Combinatorics

Mid-Term Examination

Instructions: All questions carry equal marks.

1. Define *Combinatorial Geometry* and rank of a flat in a combinatorial geometry. Prove that a flat E covers a flat F if and only if

$$\text{rank}(E) = \text{rank}(F) + 1.$$

2. State and prove the semi-modular law in combinatorial geometry.

3. State Paasch Axiom. Let V be an n-dimensional vector space over a finite field $\mathbb{F}_q$. Prove that the incidence structure whose points are k-dimensional subspaces of V and lines are $k+1$-dimensional subspaces of V satisfy the Paasch Axiom.

4. Define a $t-(v,k,\lambda)$ design. Prove that any such design is also a $i-(v,k,\lambda')$ design for all $0 \leq i \leq t$ by computing the value of λ'.

5. Prove that in any non-trivial Steiner system $S(t,v,k)$, we must have

$$v \geq (t+1)(k+1-t).$$